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LETTER TO THE EDITOR

Universal ratios of scaling amplitudes in the Hamiltonian limit of the 3D Ising model

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Abstract. The scaling amplitudes of the spin–spin and energy–energy correlation lengths for the Hamiltonian limit of the 3D Ising model are computed on a square lattice. Both periodic and antiperiodic boundary conditions are considered. The ratios of the scaling amplitudes are found to be universal.

In this letter, we study the finite-size scaling amplitudes of the spin–spin $(\sigma\sigma)$ and the energy–energy $(\varepsilon\varepsilon)$ correlation lengths of the Hamiltonian (Henkel 1984):

$$H = -h\sum_{n}\sigma^{z}(n) - \tfrac{1}{2}\sum_{n,n'}\left[(1+\eta)\sigma^{x}(n)\sigma^{x}(n') + (1-\eta)\sigma^{y}(n)\sigma^{y}(n')\right] \qquad (1)$$

on a square $N\times N$ lattice. Nearest-neighbour interactions are understood and the σ^x σ^y, σ^z are the Pauli matrices.

For $\eta=1$, H is the Hamiltonian limit of the transfer matrix of the 3D Ising model. For each $\eta\neq 0$, there is a critical point falling into the 3D Ising universality class (Henkel 1984). The critical fields are at $h_c=3.047$, 2.720 and 2.50 for $\eta=1.0$, 0.7 and 0.5, respectively.

H commutes with the operator

$$Q = \tfrac{1}{2}\left[1-\exp\left(\mathrm{i}\pi\sum_{n}\sigma^{z}(n)\right)\right] \qquad (2)$$

and thus can be written as a direct sum of block matrices. The blocks are labelled sector 0 and 1 according to the eigenvalues of Q. The inverse spin–spin correlation length is the energy gap between the lowest lying states in the two sectors and the inverse energy–energy correlation length is the gap of the ground state and the first excited state in the sector 0.

We now consider the amplitudes $A(\eta)=N\xi^{-1}$. We claim that their ratios are universal, that is, independent of η. In table 1 we show the spin–spin amplitude and in table 2 the energy–energy amplitude, both for periodic boundary conditions. In the last row, the estimated limit $(N\to\infty)$ is given as obtained from the van den Broeck and Schwartz (1979) algorithm.

In table 3, we give the energy–energy amplitude for antiperiodic boundary conditions. To obtain the limit, we use the van den Broeck and Schwartz algorithm for $\eta=1.0$ and 0.5. For $\eta=0.7$, linear extrapolation was used. Finally, in table 4, we give the spin–spin amplitude for antiperiodic boundary conditions. We use the figures for $N=4$ to estimate the amplitude $A^{(a)}_{\sigma\sigma}$.

Table 1. Scaling amplitude $A^{(p)}_{\sigma\sigma}$ for the spin-spin correlation and periodic boundary conditions.

N	$\eta = 1.0$	$\eta = 0.7$	$\eta = 0.5$
2	4.781 370	3.322 222	2.322 897
3	4.514 484	3.339 084	2.485 795
4	4.412 579	3.306 836	2.503 780
5	4.368 286	3.287 815	2.498 195
$A^{(p)}_{\sigma\sigma}$	4.334	3.260	2.500

To test for universality of the ratios of the amplitudes is equivalent to showing that all amplitudes are multiplied by a common prefactor $f(\eta)$. We use the figures of table 1 to determine f, with the normalisation $f(1) = 1$.

In table 5, we display the renormalised amplitudes $A(1)f(\eta)$. There is a clear agreement with the amplitudes $A(\eta)$ from tables 2–4.

To conclude, we have given evidence for universality of the ratios of scaling amplitudes of the spin-spin and energy-energy correlation lengths for both periodic

Table 2. Scaling amplitude $A^{(p)}_{\varepsilon\varepsilon}$ for the energy-energy correlation and periodic boundary conditions.

N	$\eta = 1.0$	$\eta = 0.7$	$\eta = 0.5$
2	17.280 725	13.108 967	10.365 889
3	16.762 609	12.653 830	9.789 510
4	16.413 195	12.368 032	9.523 306
5	16.187 071	12.194 342	9.371 803
$A^{(p)}_{\varepsilon\varepsilon}$	15.77	11.93	9.172

Table 3. Scaling amplitude $A^{(a)}_{\varepsilon\varepsilon}$ for the energy-energy correlation and antiperiodic boundary conditions.

N	$\eta = 1.0$	$\eta = 0.7$	$\eta = 0.5$
2	12.188 000	10.880 000	10.000 000
3	37.415 321	31.671 688	27.998 591
4	38.555 257	31.119 567	26.219 228
5	38.861 530	30.617 931	25.063 517
$A^{(a)}_{\varepsilon\varepsilon}$	38.97	28.6	22.9

Table 4. Scaling amplitude $A^{(a)}_{\sigma\sigma}$ for the spin-spin correlation and antiperiodic boundary conditions.

N	$\eta = 1.0$	$\eta = 0.7$	$\eta = 0.5$
2	6.094 000	5.440 000	5.000 000
3	13.345 778	10.597 133	8.566 837
4	14.081 029	10.576 150	8.166 725
$A^{(a)}_{\sigma\sigma}$	14.1	10.6	8.17

Table 5. Renormalised amplitudes $A(1)\,f(\eta)$ and the prefactor $f(\eta)$.

	$\eta = 1.0$	$\eta = 0.7$	$\eta = 0.5$
$A^{(p)}_{\varepsilon\varepsilon}$	15.77	11.86	9.10
$A^{(a)}_{\varepsilon\varepsilon}$	38.97	29.3	22.5
$A^{(a)}_{\sigma\sigma}$	14.1	10.6	8.14
$f(\eta)$	1.000	0.752	0.577

and antiperiodic boundary conditions in the Hamiltonian limit of the 3D Ising model, with the Hamiltonian defined on a square lattice.

This result supports the hyperuniversality hypothesis of Privman and Fisher (1984), which states that the correlation lengths should read

$$\xi_i = NS_i(C_1 g_1 N^{1/\nu}, C_2 g_2 N^{\Delta/\nu}) \tag{3}$$

where $g_1 = (h - h_c)/h_c$, $g_2 = b$ is a magnetic field, C_1 and C_2 are non-universal system-dependent constants and S_i is a universal function. The index i labels the different correlation lengths (e.g. $i = \sigma\sigma$ or $\varepsilon\varepsilon$). It is important to note that in (3) there is no further non-universal constant involved.

However, in the Hamiltonian limit of a transfer matrix, the normalisation of the Hamiltonian is arbitrary, which gives rise to another non-universal normalisation factor in (3), which is the $f(\eta)$ for the Hamiltonian of (1). Our results demonstrate the universality of $S_i(0, 0)$.

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